

# Strongly Coupled Dark Energy Cosmologies: preserving $\Lambda$ CDM success and easing low scale problems I – Linear theory revisited

Silvio A. Bonometto<sup>1,2\*</sup>, Roberto Mainini<sup>3</sup>, Andrea V. Macciò<sup>4</sup>

<sup>1</sup>Physics Department, Astronomy Unit, Trieste University, Via Tiepolo 11, I-34143 Trieste, Italy

<sup>2</sup>I.N.A.F., Osservatorio Astronomico di Trieste, I-34143 Trieste, Italy

<sup>3</sup>Physics Department G. Occhialini, Milano–Bicocca University, Piazza della Scienza 3, I-20126 Milano, Italy

<sup>4</sup>Max-Planck-Institut für Astronomie, Königstuhl 17, D-69117 Heidelberg, Germany

Accepted XXXX . Received XXXX; in original form XXXX

## ABSTRACT

In this first paper we discuss the linear theory and the background evolution of a new class of models we dub SCDEW: Strongly Coupled DE, plus WDM. In these models, WDM dominates today’s matter density; like baryons, WDM is uncoupled. Dark Energy is a scalar field  $\Phi$ ; its coupling to ancillary CDM, whose today’s density is  $\ll 1\%$ , is an essential model feature. Such coupling, in fact, allows the formation of cosmic structures, in spite of very low WDM particle masses ( $\sim 100$  eV). SCDEW models yields Cosmic Microwave Background and linear Large Scale features substantially undistinguishable from  $\Lambda$ CDM, but thanks to the very low WDM masses they strongly alleviate  $\Lambda$ CDM issues on small scales, as confirmed via numerical simulations in the II associated paper. Moreover SCDEW cosmologies significantly ease the coincidence and fine tuning problems of  $\Lambda$ CDM and, by using a field theory approach, we also outline possible links with inflationary models. We also discuss a possible fading of the coupling at low redshifts which prevents non linearities on the CDM component to cause computational problems. The (possible) low- $z$  coupling suppression, its mechanism, and its consequences are however still open questions –not necessarily problems– for SCDEW models. The coupling intensity and the WDM particle mass, although being extra parameters in respect to  $\Lambda$ CDM , are found to be substantially constrained *a priori* so that, if SCDEW is the underlying cosmology, we expect most data to fit also  $\Lambda$ CDM predictions.

**Key words:** large-scale structure of Universe, dark matter, dark energy - Galaxy: evolution, formation

## 1 INTRODUCTION

When SNIa Hubble diagrams revitalized  $\Lambda$ CDM, one could hardly guess how successful this cosmology would be in meeting cosmological data. In spite of its coincidence and fine tuning problems, therefore,  $\Lambda$ CDM is surely the benchmark for any attempt to improve our understanding of cosmology.

In this series of works we present a detailed analysis of a new class of cosmologies that preserves all successes of  $\Lambda$ CDM models on large and intermediate scales. In turn, they improve the agreement with the observed dark matter distribution on small scales (e.g. Oh et al. 2011; Walker & Peñarrubia 2011), at the same time

easing the coincidence and fine tuning problems of  $\Lambda$ CDM, while possibly launching a bridge between the reheating stages in the late inflationary regime and today’s Dark Energy (DE) nature.

These cosmological models are based on the presence of three dark components: i) a very-warm dark matter component (WDM), uncoupled (like the baryons), and constituting the observable dark matter at the present time; ii) a scalar field  $\Phi$  which acts as Dark Energy; iii) a peculiar Cold Dark Matter component (CDM), whose coupling to  $\Phi$  played an essential role in cosmic history, but whose density at  $z = 0$  naturally became almost negligible. We dubbed these models SCDEW (Strongly Coupled DE + WDM).

The basic features of these models were discussed in two previous papers (Bonometto, Sassi, & La Vacca (2012); Bonometto & Mainini (2014); BSLV12 and BM14 respectively, hereafter). BSLV12 dealt with background components: they showed that a purely kinetic scalar field  $\Phi$  strongly coupled to a CDM component would both exhibit densities  $\propto a^{-4}$  ( $a$  :

\* E-mail: bonometto@oats.inaf.it

scale factor), during the radiative eras, expanding along a tracker solution, with (primeval) density parameters  $\mathcal{O}(0.1\%)$ . The derelativisation of a further WDM component eventually turns these components into quintessential DE and a tiny CDM component. The radiative eras of these models are therefore characterized by three low density components, in top of the *usual*  $\gamma$ 's and  $\nu$ 's: WDM, coupled CDM and scalar field, accounting for constant fractions of the overall density, and sharing similar densities. BM14 then deals with fluctuation evolution, showing that CMB anisotropy and polarization spectra, in these models, are hardly distinguishable from the  $\Lambda$ CDM benchmark, while coupled CDM fluctuations continue to grow also between their entry in the horizon and matter–radiation equality, being so able, in spite of the low CDM density, to revitalise WDM fluctuations on scales suffering an early free streaming, even when the WDM particle mass  $m_w$  is quite small.

The  $\Phi$ –CDM coupling is therefore an essential feature in the early model evolution. After WDM has derelativized, instead, the  $\Phi$ –CDM coupling is unessential, while leading to technical difficulties, as  $\delta_c$  (CDM fluctuation amplitudes) tend to approach unity, developing then some early non-linearities, typically involving  $\ll 1\%$  of the total mass. However, switching off the  $\Phi$ –CDM coupling after the break of “conformal invariance”, yields more comfortably tractable models.

The plan of the paper is as follows. In the next Section we shall formulate the Lagrangian theory for strongly coupled Dark Energy cosmologies. It confirms the prediction on the coupled  $\Phi$ –CDM component densities, while outlining possible links with inflationary theories. In Section 3 we discuss the evolution of background parameters, focusing on the exit from the primeval stationary regime. We also widen the range of models in respect to BSLV12 and BM14 papers, by allowing for  $\beta$  fading after the exit from conformally invariant expansion. In Section 4 we discuss fluctuation evolution, taking also into account a possible  $\beta$  fading. The effects of the early growth of CDM fluctuations are therefore analysed, both as cause for the restart of WDM and baryon fluctuations, and for the possible formation of late CDM structures. We also show that, in order to mimic  $\Lambda$ CDM phenomenology through SCDEW models, the mass of WDM particles and the coupling are significantly constrained. In Section 5 fluctuation spectra are obtained and the model for N–body simulations of Paper II is conveniently selected. The last Discussion Section outlines the predictions of SCDEW models, namely for what concerns their discrimination from  $\Lambda$ CDM, whose main results are however faithfully reproduced. We also outline how SCDEW cosmologies, besides of easing low–scale  $\Lambda$ CDM conundrums, free us from its coincidence paradox and fine tunings.

## 2 COUPLED DARK ENERGY IN THE EARLY UNIVERSE

There are direct evidences that Dark Matter (DM) is a physical cosmic component, mostly clustering with observable baryons. Doubts have been cast, on the contrary, on the true nature of Dark Energy (DE); here we assume it to be a self–interacting scalar field  $\Phi$ , keeping essentially unclustered. In this scheme, both dark components interact with baryons and radiations just gravitationally, so that the energy *pseudo*–conservation equation

$$T^{(c)}_{\nu;\mu} + T^{(\Phi)}_{\nu;\mu} = 0 \quad (1)$$

holds (here  $T_{\mu\nu}^{(c,d)}$  are the stress–energy tensors of DM and DE, their traces reading  $T^{(c,\Phi)}$ ). This sum can vanish thanks to separate vanishings of both terms. The alternative option that

$$T^{(\Phi)}_{\nu;\mu} = +CT^{(c)}_{\nu;\mu}, \quad T^{(c)}_{\nu;\mu} = -CT^{(\Phi)}_{\nu;\mu}, \quad (2)$$

however, is also widely considered in the literature (see, e.g., Ellis *et al.* (1989); Wetterich (1995); Amendola (1999); Amendola & Tocchini-Valentini (2002); Macciò *et al.* (2004); Baldi *et al* (2010)), together with other possible options for energy transfer between DM and DE (see, e.g., Lopez Honorez *et al.* (2010)). The coupling

$$C = b/m_p = (16\pi/3)^{1/2}\beta/m_p, \quad (3)$$

sets the intensity of the energy flow from DM to DE. The option of scale dependent  $C$  is also discussed in the literature, namely in connection with specific models (see, e.g., R. Mainini & S.A. Bonometto 2004). Let the background metric then read

$$ds^2 = a^2(\tau)(d\tau^2 - d\lambda^2), \quad (4)$$

( $\tau$ : conformal time,  $d\lambda$ : the spatial element); eqs. (2) then yields

$$\ddot{\Phi} + 2\frac{\dot{a}}{a}\dot{\Phi} = -a^2V' + Ca^2\rho_c, \quad \dot{\rho}_c + 3\frac{\dot{a}}{a}\rho_c = -C\rho_c\dot{\Phi}, \quad (5)$$

$\rho_c$  being the DM density, while  $V(\Phi)$  is a self–interaction potential, as is required within quintessential DE models.

The rational is to allow for an energy flow from (cold)–DM to DE. In this way, the field density could keep a steady fraction (some permils) of CDM density during the whole cosmic expansion, in spite of  $\Phi$  being essentially kinetic above a suitable redshift  $z_\pm$ . At  $z_\pm$  the DE state parameter therefore shifts from  $\sim -1$  to  $\sim +1$  and such shift is found to be a generic feature, independently of the shape of  $V(\Phi)$ . By keeping a significant field density at high  $z$ , this option eases one of the coincidence problems of  $\Lambda$ CDM.

### 2.1 An early $\Phi$ –CDM coupling

Within a field theory context, a possible assumption is that CDM is a non–relativistic Dirac spinor field  $\psi$ , interacting with  $\Phi$  through a Yukawa–like lagrangian

$$\mathcal{L}_m = -\mu f(\Phi/m)\bar{\psi}\psi; \quad (6)$$

here 2 mass scales,  $m = m_p/b$  and  $\mu = g m_p$  are introduced for dimensional reasons,  $m_p$  being the Planck mass. In particular,  $b$  coincides with the coupling parameter in eq. (3) (see below).

By assuming a kinetic part of the scalar field lagrangian  $\mathcal{L}_k \sim \partial_\mu\Phi\partial_\mu\Phi$ , its equation of motion reads

$$\ddot{\Phi} + 2\frac{\dot{a}}{a}\dot{\Phi} = -a^2V' - a^2\rho_c\frac{f'}{f}, \quad (7)$$

once we work out that

$$\rho_c = -\mathcal{L}_m = \mu f(b\Phi/m_p)\bar{\psi}\psi, \quad (8)$$

in the absence of a significant kinetic term for the spinor quanta. Notice that the number density operator for the spinor field  $n \propto \bar{\psi}\psi$ , so that

$$\rho_c \propto f(\Phi/m)a^{-3} \quad (9)$$

according to the findings of Das *et al.* (2006).

Eq. (7) is consistent with the former eq. (5) if  $f'/f = -b/m_p$  so that, if the function inserted in the Lagrangian (6) has the form

$$f = \exp(-b\Phi/m_p) \quad (10)$$

we re-obtain the coupled-DE theories of eqs. (2). Also  $\rho_c$ , according to eq. (9), then exhibits a sort of exponential scaling, unless we assume that the argument of the exponential

$$b\Phi/m_p = \ln(\tau/\tau_r) , \quad (11)$$

$\tau_r$  being a (unconstrained) reference time; then  $f = \tau_r/\tau$  and

$$\dot{\Phi} = \frac{m_p}{b\tau} . \quad (12)$$

Let us consider the radiative era,  $\Phi$  bearing essentially kinetic energy; then the r.h.s. of eq. (7) reads  $a^2\rho_c b/m_p$ , while the whole equation means that  $\rho_c = (m_p/a\tau)^2$  (thence  $\rho_c \propto a^{-4}$ !) or, equivalently,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} \frac{1}{m_p^2} a^2 (2\beta^2 \rho_c) . \quad (13)$$

By comparing this equation with the Friedmann eq. , we deduce a constant early density parameter for CDM

$$\Omega_c \equiv \frac{\rho_c}{\rho} = \frac{1}{2\beta^2} . \quad (14)$$

In turn,  $\Phi$  being purely kinetic, owing to eq. (12), its energy density

$$\rho_\Phi \equiv \frac{\dot{\Phi}^2}{2a^2} = \left(\frac{m_p}{b}\right)^2 \frac{1}{2a^2\tau^2} = \frac{\rho_c}{2} , \quad (15)$$

coinciding with its pressure. The state parameter of the future quintessential field is then  $w = +1$ , while its *early* density parameter

$$\Omega_\Phi = \frac{\Omega_c}{2} = \frac{1}{4\beta^2} . \quad (16)$$

In connection with the scale dependence found for  $\rho_c$ , let us outline that, if we read  $\mathcal{L}_m$  as a mass term of the Dirac spinor, its mass scales  $\propto a^{-1}$  (as though being *redshifted*). As a matter of fact, both  $\Phi$  and CDM energy densities scale as  $a^{-4}$ . In the absence of coupling they would scale as  $a^{-6}$  and  $a^{-3}$ , respectively. The flow of energy from CDM to  $\Phi$  is tuned to yield a faster (slower) CDM ( $\Phi$  field) dilution.

The key issue, however, shown in BSLV12, is that these conditions are an attractor: even significantly perturbing “initial conditions”, the cosmic evolution rapidly settles on the behavior here described. Accordingly, such kind of evolution could last since a very early epoch, even since the late inflationary stages.

We might tentatively assume that cosmic reheating was due to the very  $\Phi$  field decaying into  $\psi$  field quanta, a process stopping as soon as the attractor solution is attained. Interactions of  $\psi$  with other fields would then allow the  $\Phi$  field energy to reheat other cosmic components, including both  $\gamma$ 's &  $\nu$ 's, as well as any other component then belonging to the *thermal soup*.

The ensuing self-similar expansion, that can be defined *conformally invariant* (in the sense discussed, e.g., by Parker 1969, 1971), can only be broken if a component scaling differently from  $a^{-4}$  achieves a significant density. If the attractor solution is followed since inflation,  $\Phi$  has had just a logarithmic time dependence since then. Its today's value, therefore, is just  $\sim 60$  times its value at the end of the reheating stages.

### 3 EXIT FROM THE STATIONARY REGIME

In any reasonable cosmological model, baryon density would eventually break the conformally invariant expansion at a redshift

$z_b \gtrsim 500$ . Adding just a baryon component to radiative components (coupled and uncoupled) is however insufficient to meet observations. Among viable possibilities, BM14 outlined the option of including a Warm DM (WDM) component of thermal origin, with a temperature parameter  $T_w$ . Its derelativization occurs when  $T_w \sim m_w$  (mass of WDM quanta) at a redshift  $z_w$ . At  $z \gg z_w$ , being  $P_w \simeq \rho_w/3$ , it is  $\rho_w \propto T_w^4$  and WDM is one of the components of the *thermal soup*; then, at  $z \ll z_w$ , being  $P_w \ll \rho_w$  and therefore  $\rho_w \propto T_w^3$ , WDM shall overcome the radiative component density so that the early stationary regime reaches an end.

This also sets an end to the attractor solution of the coupled components. During their successive evolution, as in any model allowing for CDM-DE coupling with small or large  $\beta$ , the state parameter of the field component must turn from  $+1$  to  $\simeq -1$ . This must occur *about a suitable redshift*  $z_\pm$  so to allow a fair amount of today's DE. In any approach based on a self interaction potential  $V(\Phi)$ , the fair  $z_\pm$  value is obtained by tuning some parameter(s) inside  $V(\Phi)$  expression itself. SCDEW models behave similarly. It is just convenient to make use of the first order field equation

$$\dot{\Phi}_1 + \tilde{w} \frac{\dot{a}}{a} \Phi_1 = \frac{1+w}{2} C \rho_c a^2 , \quad (17)$$

instead of eq. (5); here  $\Phi_1 \equiv \dot{\Phi}$  and  $2\tilde{w} = 1+3w-d\log(1+w)/d\log a$ . This equation, shown by BSLV12, requires  $w(a)$  to be assigned, instead of a potential  $V(\Phi)$ . In this way, we select  $z_\pm$  directly, without arguing about untestable  $V(\Phi)$  expressions. More explicitly, let us assume that

$$w = \frac{1-A}{1+A} \quad \text{with} \quad A = \left(\frac{a}{a_\pm}\right)^\epsilon \quad (18)$$

with  $a_\pm = (1+z_\pm)^{-1}$ . In BM14 we also discussed the (minimal) dependence of results on the parameter  $\epsilon$  (= 2.9 here), whose arbitrariness replaces the choice of  $V(\Phi)$  expression.

An option, not mentioned by BSLV12 and BM14, is that  $\beta$  gradually fades, after the rise of  $\rho_w$  broke conformal invariance. No wanted feature of this class of cosmologies really depends on a late coupling and we shall see that, if  $\beta$  fading occurs late enough, just minor quantitative changes arise.

Let us then consider the option that  $\beta \propto \exp(-a/a_{dg})$  with  $a_{dg} = D \times a_w$ . If we expect  $\beta$  fading to be a sort of *consequence* of the loss of conformal invariance, it should be  $D \gg 1$ . Accordingly, different options will be labelled by the value of the delay (del.)

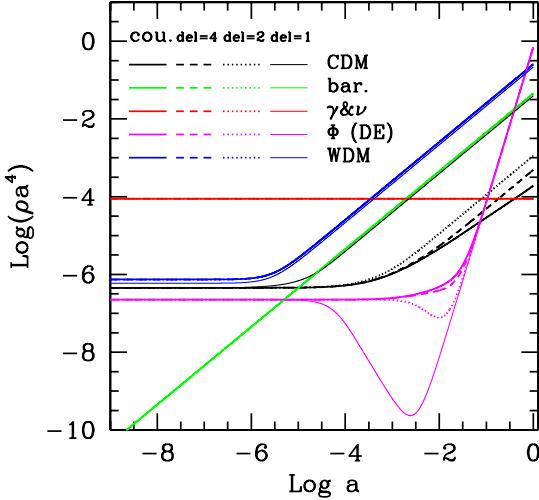
$$d = \log_{10} D \quad (19)$$

which is an extra parameter we introduce here. In Figure 1 we show the scale dependence of cosmic components in 4 different cases, passing from the option of ever lasting coupling, down to the case of delay  $d = 1$ . In this Figure and anywhere in paper I, we select the following parameter values:

$\Omega_{0\Phi}$	$\Omega_{0b}$	$h_0$	$T_{CMB}$	$N_\nu$	$n_s$
0.7	0.045	0.685	2.726	3.04	0.968

Here  $\Omega_{0\Phi}$ ,  $\Omega_{0b}$ ,  $h_0$ ,  $T_{CMB}$ ,  $N_\nu$ ,  $n_s$  are the present DE and baryon density parameters, the present Hubble parameter in units of 100 (km/s)/Mpc, the CMB temperature, the number of (almost) massless  $\nu$ 's, the primeval scalar fluctuation index, respectively. In the list,  $\Omega_{0w}$  and  $\Omega_{0c}$  are not included. Neglecting  $\gamma$ 's and  $\nu$ 's, their sum is  $1 - \Omega_{0\Phi} - \Omega_{0b}$ , while  $\Omega_{0,c}$  is fixed by the selection of the coupling constant  $\beta$  and  $\Omega_{0w}$  covers the residual gap. Notice that, at  $z = 0$ ,  $\Omega_{0,c} \ll 1/2\beta^2$  being, typically,  $\sim 10^{-2}\Omega_{0b}$  (see Figure 1).

The option  $d = 4$ , that will be selected in Paper II, is characterized by  $a_{dg} \simeq a_\pm$ ; as shown in Figure 1, for  $d = 4$  density evolution is just marginally affected by the fading of coupling.



**Figure 1.** Scale dependence of the densities of cosmic components after the break of conformal invariance. All models in this plot have  $\beta = 10$ ,  $m_w = 90$  eV. Delay (del) values from 1 to  $\infty$  (ever lasting coupling) are considered. The thick curves labelled “cou” concern an ever lasting coupling. The behavior of densities does not suffer major changes, in respect to this option, unless  $d < 2$ .

An earlier decoupling, instead, bears a number of consequences. Among them, a rise of today’s CDM density parameter  $\Omega_{c,0}$ , because the energy leaking from CDM to DE has a stop. If keeping  $d \gtrsim 2$ , this rise is limited and, in this range of  $d$  values, the present CDM density parameter  $\Omega_{c,0} \simeq 10^{-2} \times \Omega_{b,0}$ : the present CDM contribution to the cosmic budget is  $\sim 1/100$  of baryons.

A related effect is the formation of a dip in  $\rho_\Phi a^4$  evolution. As a matter of fact, until  $\Phi$  is purely kinetic, its density keeps  $\rho_\Phi = m_p^2/(2b a^2 \tau^2)$ , thanks to the energy inflow from CDM. In the radiative era, when  $\tau \propto a$ ,  $\rho_\Phi a^4$  is then constant. The transition to matter dominance should strengthen  $\rho_\Phi$  decline; however, when this occurs, coupled CDM density starts to increase, so providing a stronger  $\Phi$  feeding. If the feeding stops before the kinetic–potential transition, occurring at  $z_\pm$ , a progressive dilution of  $\rho_\Phi$  is unavoidable. However, this produces a significant dip only if  $d < 2$ . Owing to these reasons, in the rest of this Paper I we shall never consider options  $d < 2$ . We shall also verify that models with any  $d > 2$  exhibit just minor differences.

All through this discussion, we deliberately refrained from introducing any detailed physics causing a  $\beta$  fading. Possible mechanisms are discussed in Appendix A and further options surely exist, but the introduction of a further parameter can hardly be avoided. The treatment given in this work keeps on the phenomenological side, just showing that  $\beta$  fading can be self consistently introduced, so allowing a plain treatment of nonlinear stages.

Early nonlinear evolution will be expected to induce a hierarchical rippled CDM distribution, made of “virialized” structures, on growing scales.

When and if  $\beta$  fades, CDM turns into a component of fast heavy particles. We expect them to cause no substantial effect on the linear evolution of other components, also because of the smallness of the CDM density parameter; on this ground we decided to not consider these effects in our linear treatment.

Before concluding this discussion on background evolution let us outline that the relation between expansion factor  $a$  (or redshift) and ordinary time  $t$  are almost identical in SCDEW and  $\Lambda$ CDM

models; discrepancies never exceed 0.01% for any values of coupling  $\beta$  and delay  $d$ .

#### 4 LINEAR FLUCTUATION EVOLUTION

In BM14 fluctuation evolution is studied in detail. Out of horizon *initial conditions* are determined and the system of differential equations, holding after the entry into the horizon, is numerically solved. Here, let us only remind some peculiar aspects, that this problem does not share with similar cosmological models. In particular, field fluctuations are conveniently described by a variable  $\varphi$ , related to the quintessential field  $\phi$  as follows:

$$\phi = \Phi + \frac{b}{m_p} \varphi, \quad (20)$$

$\Phi$  being the background field. At the first perturbative order  $\varphi$  fulfills the equation

$$\ddot{\varphi} + 2\frac{\dot{a}}{a}\dot{\varphi} + \dot{\Phi}\dot{h} + k^2\varphi + a^2V''(\Phi)\varphi = 2\beta^2\Omega_c \left(\frac{\dot{a}}{a}\right)^2 \delta_c. \quad (21)$$

Derivatives are taken in respect to the conformal time, as the metric reads

$$ds^2 = a^2(\tau)[d\tau^2 - (\delta_{ij} + h_{ij})dx_i dx_j]. \quad (22)$$

Moreover,  $\delta_c$  are the fluctuations in the coupled CDM component and  $h$ , the trace of the 3-tensor  $h_{ij}$ , describes gravity. Finally,  $k$  yields the mass scale  $M = (4\pi/3)\rho(2\pi/k)^3$  of the fluctuation.

Most terms in eq. (21) have therefore a transparent meaning, apart of the term  $V''(\Phi)$ , where we apparently refer to a  $\phi$  self-interaction potential. Aiming to skip any reference to such quantity, BM14 show that one can use the relation

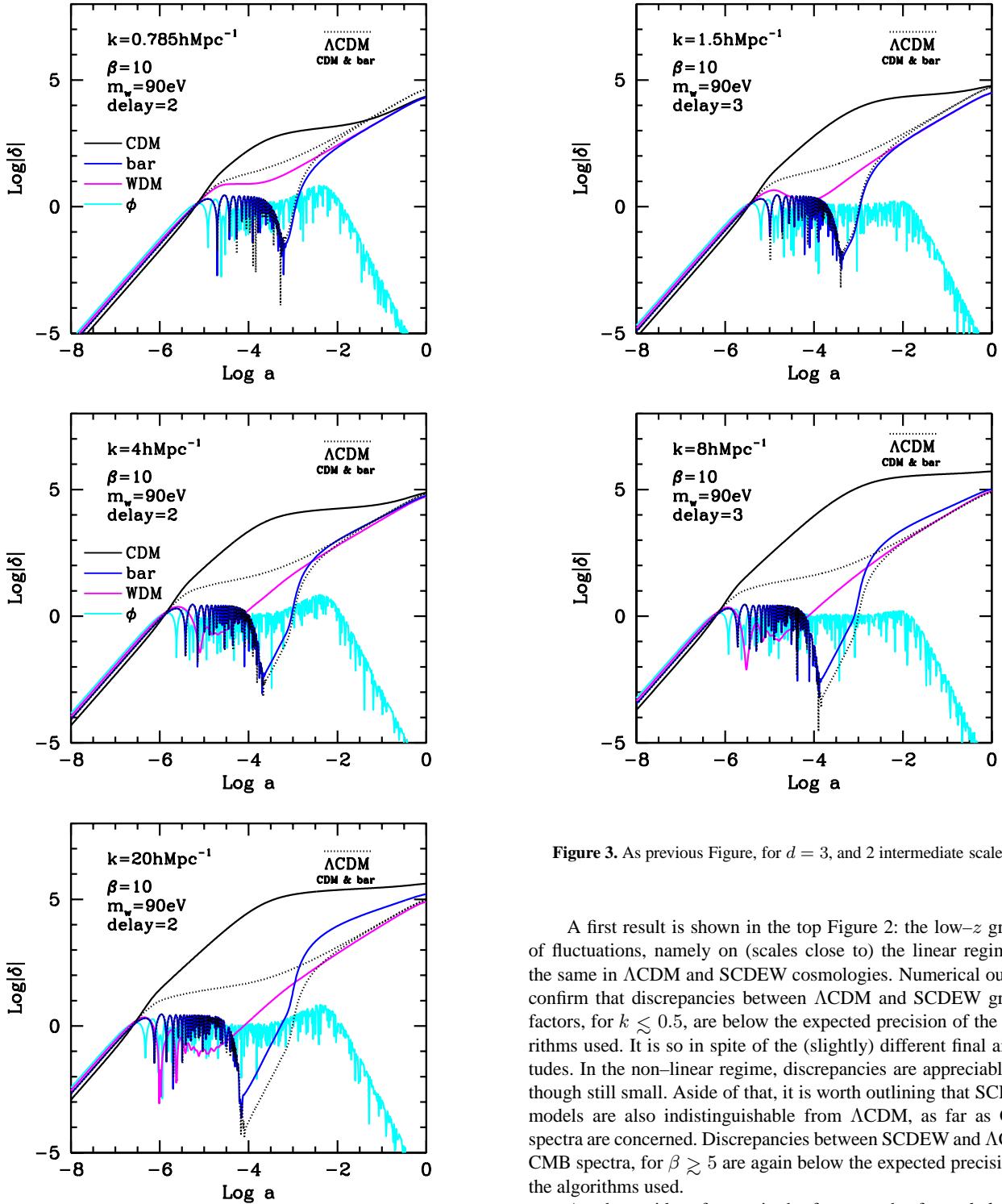
$$2V'' = \frac{A}{1+A} \left\{ \frac{\dot{a}}{a} \frac{\epsilon}{1+A} \left[ \epsilon_6 \frac{\dot{a}}{a^3} + 2C \frac{\dot{\rho}_c}{\dot{\Phi}} \right] + \left[ \frac{\dot{a}}{a^3} \frac{\ddot{\Phi}}{\dot{\Phi}} + \frac{d}{d\tau} \left( \frac{\dot{a}}{a^3} \right) \right] \epsilon_6 + 2C \frac{\dot{\rho}_c}{\dot{\Phi}} \right\} \quad (23)$$

with  $\epsilon_6 = \epsilon - 6$ . Here,  $A$  and  $\epsilon$  are the quantities defining the  $w(a)$  behavior in eq. (18). All variables in this expression are known at each step of a linear evolutionary algorithm.

It is then significant that, within this formulation, also in the fluctuation equations only  $\dot{\Phi}$  and  $\ddot{\Phi}$  appear, while  $\Phi$  values never matter. BM14 made use of these relations to modify the public algorithm CMBFAST and here we shall report results obtained with this code.

#### 4.1 Early growth of coupled CDM fluctuations

Figure 2 and 3 then show a few examples of linear evolution, for fluctuations in the different cosmic components, from before their entry in the horizon, down to  $z = 0$ . The background cosmic parameters are the same of Figure 1. The mass of the warm component is anywhere 90 eV, its range being fixed by the arguments discussed below, in the next subsection. The  $k$  values selected for Figure 2, where the delay parameter is  $d = 2$ , correspond to the mass scales of  $1.8 \times 10^{14} h^{-1} M_\odot$  (or  $8 h^{-1} \text{Mpc}$ , top panel),  $1.4 \times 10^{12} h^{-1} M_\odot$  (central panel), and  $8.7 \times 10^{10} h^{-1} M_\odot$  (lower panel). Figure 3 is for 2 intermediate scales and  $d = 3$ . For the sake of comparison, the evolution of baryon and CDM fluctuations in a  $\Lambda$ CDM model with the same cosmological parameters is also overplotted.



**Figure 2.** Evolution of density fluctuations in the model indicated in the frame for 3  $k$  values; from top to bottom they correspond to masses  $\sim 1.8 \times 10^{14} - 1.35 \times 10^{12} - 8.7 \times 10^{10} h^{-1} M_\odot$ , respectively. The first  $k$  value then corresponds to a comoving length scale of  $8 h^{-1} \text{Mpc}$ , at the boundaries between linear and non-linear behaviors; the second and third  $k$  values lay at the top and bottom limits of the galactic mass scales. Dotted curves show the evolution of CDM and baryons, for the same scales, in a  $\Lambda\text{CDM}$  model.

**Figure 3.** As previous Figure 2, for  $d = 3$ , and 2 intermediate scales.

A first result is shown in the top Figure 2: the low- $z$  growth of fluctuations, namely on (scales close to) the linear regime, is the same in  $\Lambda\text{CDM}$  and SCDEW cosmologies. Numerical outputs confirm that discrepancies between  $\Lambda\text{CDM}$  and SCDEW growth factors, for  $k \lesssim 0.5$ , are below the expected precision of the algorithms used. It is so in spite of the (slightly) different final amplitudes. In the non-linear regime, discrepancies are appreciable, although still small. Aside of that, it is worth outlining that SCDEW models are also indistinguishable from  $\Lambda\text{CDM}$ , as far as CMB spectra are concerned. Discrepancies between SCDEW and  $\Lambda\text{CDM}$  CMB spectra, for  $\beta \gtrsim 5$  are again below the expected precision of the algorithms used.

Another evident feature is the fast growth of *coupled* CDM fluctuations, not discontinued at the time when the horizon attains the fluctuation size. In  $\Lambda\text{CDM}$ , *uncoupled* CDM fluctuation grow much more slowly, namely until radiation exceeds CDM density, as radiation fluctuations (still coupled to baryons) have stopped their growth, being in the sonic regime. In SCDEW models, this extra growth is expected, in spite of the small density  $\rho_c \simeq \rho_t/2\beta^2$  ( $\rho_t$ : total density) of the coupled CDM component: in the non-relativistic regime, coupling causes an increase of the *effective* CDM-CDM gravity by a factor  $1 + (4/3)\beta^2$  (Amendola 1999);

for  $\beta^2 \gg 3/4$ , this factor erases the division by  $2\beta^2$  so that, as far as self-gravity is concerned, it is as thought CDM had a density close to the total density  $\rho_t$ . In addition, CDM particle velocities are enhanced by a continuous push due to their progressive mass decrease (the so-called antidrag term). All these effects are taken into account by the linear program and cause an increase of CDM density fluctuations, after their entry in the horizon and in the radiation dominated epoch, approximately  $\propto a^{3/2}$ .

## 4.2 WDM fluctuation regeneration

Large amplitude CDM waves cause a restart of fluctuations in WDM, as soon as it becomes non-relativistic. As is also visible in Figures 2 and 3, WDM derelativizes at  $z_{w,der} \sim 10^5$ . Derelativization is linked to the value taken by the  $T_w/T_\gamma$  ratio, between WDM and photon temperatures. If WDM particles decoupled from the *thermal soup* when its density was  $(\pi^2/30)g_{w,dec}T^4$ , today  $(T_w/T_\gamma)^3 \simeq (2/g_{w,dec})(1 + 3N_\nu/22)$  ( $N_\nu$ : number of (almost) massless  $\nu$ 's in the cosmic background;  $g_{w,dec}$ : number of relativistic degrees of freedom at the time of WDM decoupling), while

$$\Omega_{0w}h^2 \simeq \frac{45\zeta(3)}{4\pi^4} \left(\frac{T_w}{T_\gamma}\right)^3 \frac{m_w}{10^4 T_\gamma}. \quad (24)$$

For  $N_\nu = 3$  we have then

$$\Omega_{0w}h^2 \simeq 0.115 \frac{m_w/\text{eV}}{g_{w,dec}} \quad (25)$$

so that  $T_w/T_\gamma \sim 0.32$  if  $\Omega_{w,0}h^2 \simeq 0.12$  and  $m_w \simeq 90$  eV. Accordingly,  $T_w \simeq m_w$  at a redshift  $\sim 10^5$ – $10^6$ , in agreement with what is shown in the plots, these effects being all included in modified CMBFAST. In particular, our algorithm takes into account the contributions of coupled CDM and  $\Phi$  field to the *thermal soup*.

At the time of WDM derelativization, CDM fluctuations  $\delta_c$  exceed then any other fluctuations, by a scale dependent factor  $F_b \sim 10^2$ – $10^3$  for galactic scales. There is no boost to the CDM–WDM gravity due to coupling, but the CDM density excess  $\delta\rho_c = \rho_c\delta_c \simeq (\rho_t/2\beta^2)\delta_c$  is boosted by the factor  $F_b$ , compensating the division by  $2\beta^2 \sim 10^2$  due to the small CDM density. As the factor  $F_b$  depends on the time elapsed since the entry in the horizon, the restart of WDM fluctuations should be more and more effective towards greater  $k$  values. In turn, WDM particle escape velocity from smaller size fluctuations is smaller, so that the effect is only partially visible in the final spectra.

On the contrary, this very effect is unchallenged in baryons, whose mean velocities are negligible. Also the (later) restart of their fluctuations is mostly due to CDM. In the absence of residual particle velocities, baryon fluctuations growth even exceeds WDM fluctuations, both at high  $z$  and at  $z = 0$ , namely at high  $k$ .

Since  $\sim z_{w,der}$ , WDM density starts to dilute just  $\propto a^{-3}$ , and primeval conformal invariance is broken. Also baryon density growth violates such invariance, of course, but baryon density overcomes radiative components at a redshift  $< z_{w,der}$ . In principle, it can make sense that a break of conformal invariance and  $\beta$  fading are related events: when WDM has turned non relativistic, the  $\beta$  coupling appears like a residual pleonasm. Leaving apart detailed options to model a relation between these effects, we just quantify it with the parameter defined in eq. (19). For instance, for  $d = 2$  (3, 4),  $\beta$  fades when  $z \sim 5000$  (500, 50). Values of  $d \gg 6$  return an ever lasting  $\beta$ -coupling.

## 4.3 WDM particle mass selection

WDM fluctuation regeneration in SCDEW models contrasts with undisputed free streaming effects in standard  $\Lambda$ WDM models, causing a large- $k$  cutoff to the spectral function

$$\Delta^2(k) = \frac{1}{2\pi^2} k^3 P(k), \quad (26)$$

( $P(k) = \langle |\delta(k)|^2 \rangle$ : transferred spectrum), because of the erasing of any fluctuation entering the horizon before WDM has derelativized. With  $m_w \simeq 90$  eV, the decline of  $\Delta^2$  starts at  $k \simeq 0.5 h\text{Mpc}^{-1}$ ,  $\Delta^2$  being already damped by a factor  $\sim 10^3$  at  $k \simeq 1 h\text{Mpc}^{-1}$ . Accordingly, viable  $\Lambda$ WDM models currently refer to WDM masses  $m_w \simeq 2$ – $3$  keV, allowing fluctuations to survive up to  $k \simeq 20$ – $30 h\text{Mpc}^{-1}$ .

Unfortunately, the need of such a large  $m_w$  partially invalidates the choice of warm instead of cold DM. The residual motions of low-mass particles, e.g., are then insufficient to reduce the number of expected MW satellites or to prevent them settling on a NFW (Navarro, Frank & White 1997) profile. Recent simulations confirm a relation between  $m_w$  and the size of a core. According to Macciò *et al.* (2012), in the dwarf galaxy mass range, the size of the core

$$R_{core} \sim \left(\frac{1.0}{h^{-1}\text{kpc}}\right) \left(\frac{100 \text{ eV}}{m_w}\right)^{1.8}. \quad (27)$$

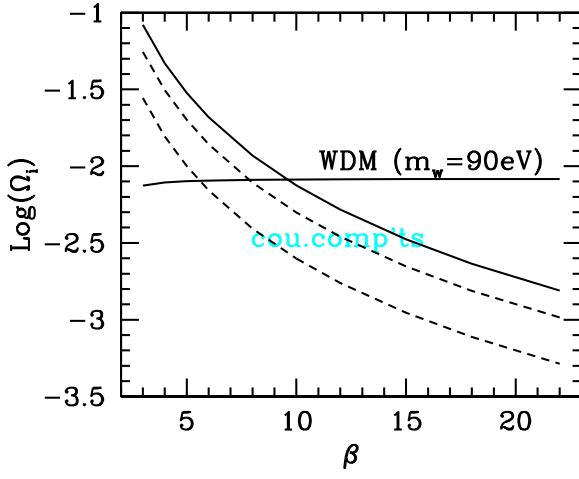
If  $m_w \sim 2$  keV, e.g., it is  $r_{core} \sim 5 h^{-1}\text{pc}$ . On the contrary, a fair core size, fitting observations, apparently requires  $m_w \sim 80$ – $110$  eV; a mass scale yielding fair cores but no galaxies. It is then significant that the linear theory of strongly coupled–DE models exhibits a full restart of fluctuations.

Before concluding that a solution to low-scale  $\Lambda$ CDM problems is found, however, we shall first consider in detail the full shape of linear spectra and use them to perform *ad-hoc* numerical simulations. The former aim is fulfilled here below, the latter one is the target of Paper II.

Meanwhile, let us outline that there are a few other elements in favor of choosing  $m_w \simeq 90$  eV, independently of the expected halo and satellite predictions. The very Figure 1 shows that the contributions of WDM, CDM and  $\Phi$  to the primeval *thermal soup* are all within half order of magnitude. Even though this is due to the choice of  $\beta$  and  $m_w$ , it would be reckless to conclude that it fixes their ranges, but it would also be hard to believe all that to be fully casual.

Figure 4 then shows a strictly related point: The primeval density parameters  $\Omega_i$  plotted there are for CDM,  $\Phi$  and WDM. The WDM density parameter exhibits just quite a mild  $\beta$  dependence. In fact, only for very low  $\beta$  values, its level is significantly eaten by a non-negligible  $3/4\beta^2$  contribute. On the contrary, the WDM level would significantly depend on  $m_w$ . Should this mass be greater by  $\Delta m$ , WDM derelativize earlier by  $\Delta a \simeq \Delta m/T_0$  and the WDM line should be lower. In order to recover the nearly-coincidence between primeval  $\Omega_i$ , fairly greater  $\beta$  values should then be selected, as the dependence on  $\beta$  is quadratic. The opposite would occur for lower  $m_w$  values, with the risk to approach the limits of the physical range for  $\beta$ 's.

This plot should be however taken together with a third argument, which can be made only after giving details on spectra (see Figure 8, below): equal normalizations for CMB angular spectra and  $\sigma_8$ , in  $\Lambda$ CDM and SCDEW models, should be required; this fixes a narrow  $\beta$  range, once  $m_w = 90$  eV is selected. In turn, Figure 4 returns  $m_w = 90$  eV for that  $\beta$  range. Altogether, the two constraints are close to a system of 2 algebraic equations



**Figure 4.** Primeval density parameter of WDM and coupled CDM– $\Phi$  components in the radiation dominated era, for  $m_w = 90$  eV. The upper (lower) dashed line yields CDM ( $\Phi$ ) density. The total density of the coupled components is the “parallel” solid curve. The slightly rising solid line is WDM density parameter.

with 2 unknown, although yielding softer constraints.  $\beta \simeq 10$  and  $m_w = 90$  eV are at the middle of the narrow allowed bands. If N-body simulations will confirm that this selection favours observational profiles, etc., we may say that this follows from the optimal parameter choice within the model.

#### 4.4 CDM non-linearities

Recovering (WDM or) baryon fluctuations thanks to coupled CDM action, on those scales where they had been erased, requires CDM fluctuations 100–1000 times wider than uncoupled CDM fluctuations in  $\Lambda$ CDM models. In fact, coupled CDM has a much smaller density parameter than *usual* CDM, when its action is needed. Luckily enough, such wide amplitude is an intrinsical feature of SCDEW models. In turn, this feature can cause the birth of early CDM nonlinearities. These non-linearities are anyway suppressed by the fact that CDM contributes to the cosmic budget as little as  $\sim 1$  per cent of the very baryonic component.

In SCDEW models early non linear CDM structures however form on small scales. In the radiation dominated period, after entering the horizon, coupled CDM fluctuations grow  $\propto \sim a^{3/2}$ . If  $\delta_c \sim 10^{-5}$  at the horizon redshift  $z_h$ , it shall be  $\delta_c \sim 1$  at  $z_{n.l.} \sim z_h \times 10^{-10/3}$ . Accordingly, fluctuations on scales entering the horizon at a redshift  $\sim 10^3$  times greater than equality, involving a mass  $\lesssim 10^7 h^{-1} M_\odot$  are quite likely to produce CDM non linear structures. Higher mass structures can also form up to a scale depending on the delay  $d$ .

The most immediate problem related to CDM non-linearities concerns the reliability of linear algorithms predicting fluctuation evolution. In fact, the first effect due to non linearity onset, is an acceleration of the growth rate. An estimate of the effect can be desumed from the theory of spherical growth in pure CDM models, telling us that a density contrast  $\simeq 5\text{--}6$  is attained when the linear  $\delta_c \simeq 1.0\text{--}1.1$ . Non-linearity triggers other effects, as mode mixing, a loss of Gaussianity and high velocity fields. Forming structures, in particular, shall have a shrinking radius and a global

motion, so that we can expect a substantial loss of coherence between the distribution of CDM and other cosmic components, with top spectral contribution eventually affecting wavelengths smaller than the original fluctuations  $\delta_c$ . Accordingly, no excessive effects are expected on scales never overcoming a mild non-linearity.

At an advanced stage, evolved CDM non-linearities shall then be non-linear structures embedded in an almost unperturbed continuum, taking also into account that, altogether, the CDM density parameter has reduced to  $\mathcal{O}(10^{-3}\text{--}10^{-4})$ , so that the perturbing bodies shall be relatively rare.

Let us also outline that, also after  $\beta$  fading, the linear algorithm will continue to treat CDM and baryons separately. As a consequence, the CDM fluctuation  $\delta_c$  can still overcome unity while the baryon fluctuation  $\delta_b$  is still  $\ll 1$ . However, in these conditions, CDM and baryon obey the same equations of motion and the physical variable is

$$\delta_{cb} = \frac{\Omega_c \delta_c + \Omega_b \delta_b}{\Omega_c + \Omega_b}. \quad (28)$$

Unless this is non-linear, physical non-linearities do not exist. Also if, slightly before  $\beta$  fading,  $\delta_c$  approached unity, but without badly loosing coherence, the physical post-fading variable will be  $\delta_{cb}$ ; however, even if coherence is partially lost, that part of  $\delta_c$  still coherent with  $\delta_b$  will enter a unified growth regime. Accordingly, the effect of marginal non linearities can be thought to be just a slight boost of the CDM+baryon fluctuation amplitudes. A quantitative estimate of such boost is not immediately feasible, although one must however recall that  $\Omega_c \ll \Omega_b$ , so that the very uncertainty shall not be too large.

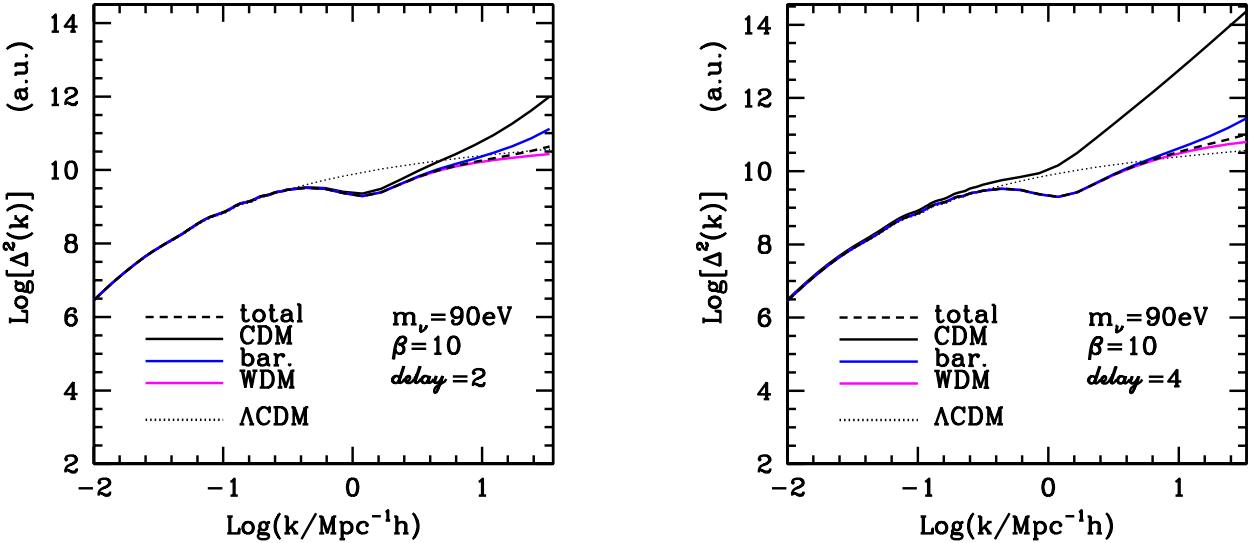
When considering the spectral function in the next Section, which are a product of linear algorithms, one must therefore discriminate among different possible *meanings* of apparently high  $\Delta^2(k)$  for the CDM component at high  $k$ .

There is however a clear conclusion for the above discussion: SCDEW models predict the formation of early non-linear CDM structures up to a mass  $\sim 10^7\text{--}10^8 h^{-1} M_\odot$ . With a present CDM density  $\sim 10^8 h^{-1} M_\odot/(h^{-1} \text{Mpc})^3$ , we expect their average comoving distance to be  $\sim 1 h^{-1} \text{Mpc}$ , a value comparable with the average intergalactic distance. Smaller residual CDM structures could also exist, if not embedded in greater structures in a hierarchical formation process. No further speculation is however possible, to better define the specific features these “objects” could exhibit, apart of the rather obvious but probably simplistic statement that they might occupy the nucleus of existing galaxies.

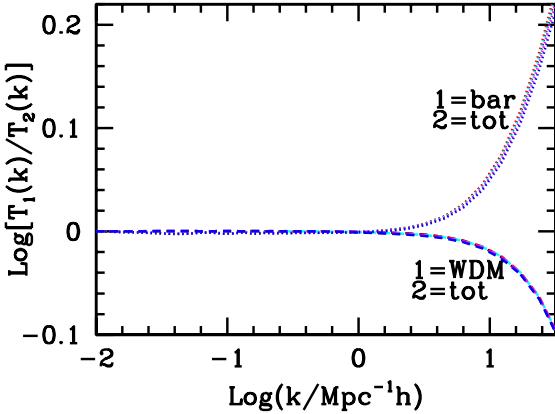
## 5 FLUCTUATION SPECTRA

The modified version of CMBFAST enables us to evaluate the transfer functions and the spectral functions  $\Delta^2(k, z)$  (see eq. 26). Figures 5 show them at  $z = 0$ , for specific models, with separate curves for different components. Results are compared with  $\Lambda$ CDM.

For  $\log k \lesssim -1$  the spectra of all cosmic components almost overlap. At greater  $k$  values, baryons and WDM exhibit a gap in respect to  $\Lambda$ CDM. In the absence of strongly coupled CDM, such hint to decline would turn into a fast decrease at slightly higher  $k$ 's, a characteristic of standard AWDM cosmologies. Here, on the contrary, thanks to the action of coupled CDM, we see a fast recovery. Baryon and WDM spectra therefore re-approach the  $\Lambda$ CDM spectrum and, in the case  $d = 4$ , overcome it. In turn, the (coupled) CDM spectra are constantly above baryon and WDM; for



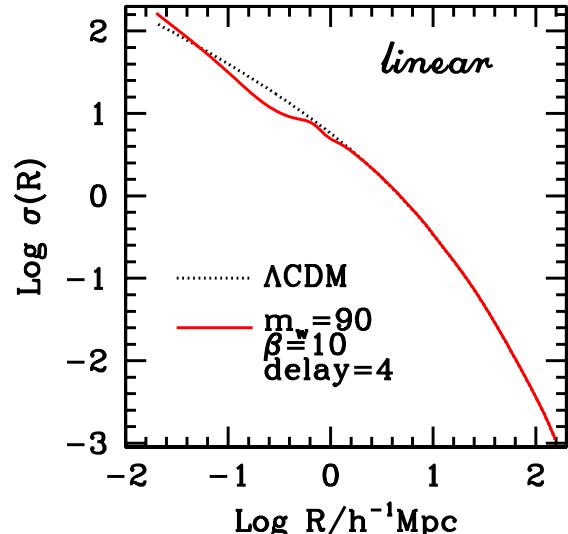
**Figure 5.** Spectral functions obtained from the linear theory at  $z = 0$  for models with  $d = 2$  and  $4$ . We compare them with  $\Lambda\text{CDM}$  spectra, finding that the *revival* of high- $k$  fluctuations is highly efficient.



**Figure 6.** High- $k$  difference between baryon and WDM transfer functions  $T(k)$ , shown to be independent from  $d$  (we overlap curves for  $d = 1.8, 2.4, 3, 3.5, 4$ ). The ratios  $T_1/T_2$  are between the transfer functions for baryon or WDM and the total transfer function, as indicated in the Figure. The model is  $m_w = 90$  eV and  $\beta = 10$ . As expected, the split starts at greater (smaller)  $k$  if a greater (smaller)  $m_w$  is taken.

$d = 2$ , however, the CDM spectrum does first decline, as baryons and WDM do, then gradually rises above them; on the contrary, for  $d = 4$ , the CDM spectrum keeps constantly above  $\Lambda\text{CDM}$  and there is just a hint of slower increase where baryons and WDM begin their gap.

The most significant point, however, is the comparison with  $\Lambda\text{CDM}$ . Figures 5 show a greater spectral discrepancy for lower  $d$  values. If we however consider  $\beta \neq 10$  values (not shown in the Figures), we have a similar evolution of CDM fluctuations, for the  $\beta$  compensation discussed in Section 4.1, but equally large CDM fluctuations are less efficient to recreate WDM and baryon fluctua-



**Figure 7.** Linear mass variance on the scale  $R$  for the model in the frame (an exponential window is used).

tions, because of the smaller CDM mass attracting them. Accordingly, the discrepancy with  $\Lambda\text{CDM}$  is even greater, in spite of CDM having spectra similar to the ones shown here. As a matter of fact, with  $\beta = 10$ , the discrepancy of baryon and WDM spectra with  $\Lambda\text{CDM}$  is not so great, never exceeding 1 order of magnitude.

There is another point, already visible in the spectral functions in Figure 5, but conveniently stressed in Figure 6: at high  $k$  the baryon spectrum starts to exceed the WDM spectrum. It is not a negligible effect. At  $k \simeq 30 h \text{ Mpc}^{-1}$ , the baryon spectrum exceeds average by  $\sim 60\%$ , while the WDM spectrum exhibits a deficiency, still in respect to average, by  $\sim 25\%$ . In Figure 6 curves

referring to models with  $d = 2, 3, 4$ , are shown to overlap. The Figure is done for  $m_w = 90$  eV; as expected, a significant increase of  $m_w$  reduces the gap.

It may be significant to see the shift between SCDEW and  $\Lambda$ CDM, when looking at the linear m.s. fluctuation

$$\sigma_R^2 = \int_0^\infty dk \Delta^2(k) W^2(kR) \quad (29)$$

(mass variance). There are different forms for the window function  $W(kR)$ ; in this work all computations were coherently performed by using an exponential window. The  $\sigma_R$  behavior is shown in Figure 7, for  $d = 4$  where, within the models considered here, it is greatest. As recalled in the frame, this is a *linear* computation, and the slight discrepancies appear on scales  $R < 1 h^{-1}$ Mpc ( $\sim 3.5 \times 10^{11} h^{-1} M_\odot$ ), surely non-linear today. The expected overall effect, after the extensive non-linear  $k$ -mode mixing, is expected to be slightly greater.

The expression (29) can be also used to study the  $\beta$  dependence of  $\sigma_8$  (mass variance on the scale of  $8 h^{-1}$ Mpc) on  $\beta$  and  $m_w$ . In Figure 8, we keep  $m_w$  fixed and study the  $\sigma_8$  dependence on  $\beta$  for models with  $d = 4$ . The main finding shown in this plot is that, in order to approach the  $\sigma_8$  value of  $\Lambda$ CDM, we need  $\beta \simeq 10$ .

Let us then outline the difference between total spectra of models differing for the  $d$  values. As shown in Figure 9, discrepancies increase with  $k$ . For  $\log k \simeq 1.5$  ( $M \simeq 2.8 \times 10^9 h^{-1} M_\odot$ ) the discrepancy is  $\sim 2$ , when we compare  $d = 4$  and 2; it becomes  $< 20\%$  when we compare  $d=3$  and 4. The above feature however exhibits just a mild  $\beta$  dependence (not shown in the Figure): for lower (higher)  $\beta$  values, the discrepancy is slightly smaller (greater).

These spectra were accurately considered to choose the SCDEW model most suitable to study non-linear effects. Coupled CDM non-linearities are safely milder with  $d = 2$ . (They would be even milder for  $d < 2$  but, as shown in Figure 1, the  $d$  dependence of the model becomes stronger and, in particular, a significant gap in  $\rho_\Phi$  behavior becomes unavoidable.) As discussed in previous Section, even in the  $d = 2$  case we expect some impact of forming CDM non-linearities, probably slightly rising the spectrum, more and more as  $k$  increases. Owing then to the limited difference between spectra, shown in Figure 9, we decided to make use of the  $d = 4$  spectrum, assuming that, probably, it best represents the  $d = 2$  case, once the upward shifts due to forming CDM non-linearities are included.

## 6 DISCUSSION

A number of experiments are in progress, aiming to inspect the nature of dark cosmic components, e.g. BOSS<sup>1</sup>, HETDEX<sup>2</sup>, DES<sup>3</sup>, LSqST<sup>4</sup> and Euclid<sup>5</sup>. Even assuming a linear expression  $w = w_0 + w_1(1 - a)$  for DE state parameter, however, errors on  $w_0$  and  $w_1$  can hardly be pushed below 10 % and some 10 %, respectively (e.g. Joachimi & Bridle 2010). Better hopes exist to detect violations of the *standard* relation between scale factor  $a$  and growth factor  $\mathcal{G}$  time dependences.

They could arise from violation of General Relativity if, e.g.,

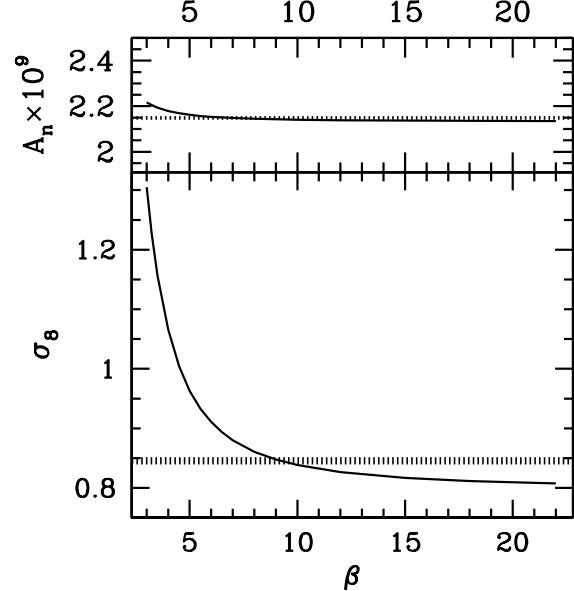
<sup>1</sup> <http://www.sdss3.org/surveys/boss.php>

<sup>2</sup> <http://hetdex.org/hetdex>

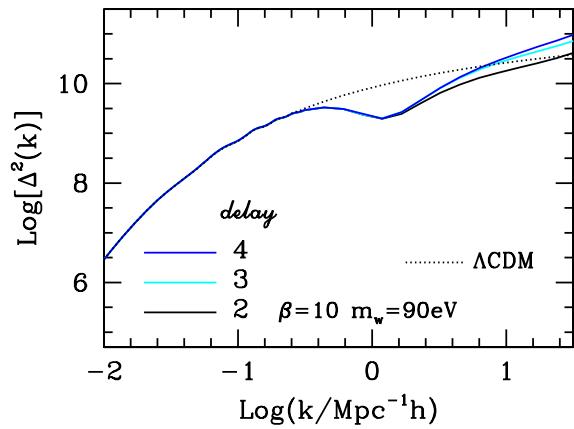
<sup>3</sup> <http://www.darkenergysurvey.org>

<sup>4</sup> <http://www.lsst.org>

<sup>5</sup> <http://www.euclid-ec.org/>



**Figure 8.** Lower plot:  $\sigma_8$  dependence on  $\beta$  for SC models best fitting CMB fluctuations (hence, normalized as shown in the top plot). In the SC model selected  $m_w = 90$  eV. Dashed horizontal lines yield normalization and  $\sigma_8$  for a  $\Lambda$ CDM model with the same parameters.



**Figure 9.** Dependence of the overall spectrum on the  $d$  parameter. At  $k \simeq 32 h \text{ Mpc}^{-1}$  ( $M \simeq 2.8 \times 10^9 h^{-1} M_\odot$ ) the ratio between  $d = 4$  and  $d = 2$  spectra is  $\lesssim 2$ .

the gravitational action  $R$  (Riemann scalar) is replaced by a suitable function  $f(R)$ . In this case the dynamics of any component, baryons and DM, would be directly modified at large distances. A large deal of work has deepened this option. N-body simulations have been also recently performed (Puchwein et al. 2013).

An alternative option is that only DM dynamics is modified, as occurs in “ordinary” coupled DE theories. Also this option has been widely debated, N-body simulations of these models have been performed since several years (Macciò et al. 2004; Baldi et al 2010; Li & Barrow 2011; Baldi 2012), finding that baryon distribution would also change, over very large scales, as an indirect consequence. Since a few years these models have been shown to

be consistent with data if  $\beta \lesssim 0.1$ , limits being widened if coupling is considered together with a non-vanishing neutrino mass, even finding a  $\sim 2\sigma$  constraint on the coupling  $\beta$  about  $\sim 0.09$  (La Vacca et al. 2009; Kristiansen et al. 2010; Pettorino et al. 2012). More recently, the coupling option was also shown to ease the tension between Planck and Hubble telescope  $H_0$  estimates, allowing Xia (2013) to turn limits into a  $> 3\sigma$  detection, yielding  $\beta = 0.078 \pm 0.022$  and, consistently,  $H_0 = 74.8 \pm 2.8$ . Also within this option, significantly affecting the dynamics of the whole DE, future data are expected to outline apparent  $a(t)$ - $\mathcal{G}(t)$  discrepancies.

Strongly Coupled cosmologies (SCDEW models), on the contrary, predict no change in  $a(t)$  or  $\mathcal{G}(t)$  in respect to  $\Lambda$ CDM and therefore, *a fortiori*, the  $a(t)$ - $\mathcal{G}(t)$  relation should be found to be consistent with  $\Lambda$ CDM predictions. The very equation of state of DE remains somehow arbitrary, although state parameters  $w < -1$  would require suitable extensions of the approach described in this work. All above experiments are therefore expected to yield results seemingly consistent with  $\Lambda$ CDM cosmologies.

This point has been accurately verified in this work. In comparison with BSLV12 and BM14, two further points were outlined here: (i) By providing a Lagrangian approach to the coupling between  $\Phi$  and  $\psi$  fields (DE and CDM, respectively) and stressing the logarithmic growth of the former one, we outlined the possibility that  $\Phi$  is both inflaton and DE. (ii) The risk of CDM fluctuations reaching an early non-linear dynamics was also outlined and *circumscribed*.

We shall further discuss the (i) point in another work. As far as the (ii) point is concerned, it is strictly linked to the *minimal* assumption of a constant interaction constant  $\beta$ . We tentatively suggested to overcome such option by admitting  $\beta$  to fade, once it accomplished its aim to allow us an (almost) conformally invariant cosmic expansion through cosmic ages. Possible mechanisms directly relating conformal invariance break to  $\beta$  fading, however, were not discussed. A number of alternative options are briefly discussed in Appendix A.

Altogether, the picture considered remains fully viable and leads to a phenomenological picture quite close to  $\Lambda$ CDM. In its theoretical framework, however, most of the unpleasant fine tunings and coincidences of  $\Lambda$ CDM are significantly eased.

SCDEW and  $\Lambda$ CDM cosmologies are however distinguishable through a number of observables: First of all, SCDEW suggests a DM particle with mass  $\sim 100$  eV. If this kind of warm-hot DM replaces the CDM of  $\Lambda$ CDM models, in the absence of strong coupling there would be almost no structure in the Universe. Even the option of mixed cold and hot-warm DM is far from fitting data on dwarf rotation curves or large galaxy satellites. Accordingly, *discovering a sterile neutrino or a gravitino in the 100 eV mass range would mark a strong point in favour of SCDEW models*.

The other critical prediction of SCDEW cosmologies is the formation of early coupled-CDM structures: Coupled-CDM fluctuations  $\delta_c$ , to be able to revive baryon and WDM fluctuations, had to be quite large when WDM particles finally became non-relativistic. This is what theory predicts and is also the only way to make their density excess  $\delta\rho_c = \rho_c\delta_c$  significant; in fact  $\rho_c \simeq 1/2\beta^2$  or less. Therefore,  $\delta_c$  approaches unity earlier than other components. If, meanwhile, the CDM- $\Phi$  coupling fades, baryons and CDM share identical equations of motion and only their total fluctuation matters. Non-linearity then becomes just a formal and harmless feature. Should it be not so, bound CDM structures form. This however occurs over mass scales  $\lesssim 10^7 h^{-1} M_\odot$  (let us how-

ever recall that, altogether, coupled CDM density is  $\sim 1\%$  of the very baryon density, today).

This risks to weaken the predictivity of linear codes. When growing non linear, CDM fluctuation gravitation becomes stronger than linear codes compute, speeding up the growth of other components. This effect is however weakened by  $\rho_c$  becoming even smaller than  $\sim 1/2\beta^2$ , as the Universe turns from radiation to matter dominated, and by the fact that other component fluctuations have recovered such a significant amplitude to yield an –at least– comparable push. Accordingly, we suggested the extra gravitational push of CDM to bear effects equivalent to shifting the parameter  $d$  from 2 to 4. Although based on qualitative arguments, this option is not unlikely to approach the real effect. Accordingly, N-body simulations in the accociated Paper II are based on the  $d = 4$  option.

All that implies a clear prediction: that early CDM primeval structures have formed. Although being extremely rare because of the low overall CDM density ( $\sim 1/100$  of baryon density), they could be the main existing structures at high redshifts ( $z \gg 10$ ) and might have an effect at later times on the formation of the first stars and/or cosmic reionization; we plan to address these issues in future work.

In our opinion, however, the strongest argument in favour of SCDEW models is the apparent easing of fine tunings. In particular, a twofold DM is an option often considered on purely phenomenological bases. However, the *true* DM, in SCDEW cosmologies, is the warm one; CDM is a sort of handyman component, fist allowing  $\Phi$  to keep on its tracker solution, then allowing inhomogeneities to revive on observational scales, eventually creating bound systems where it *hides* today. The real challenge of these models is finding out its hideouts. Apart of that, replacing  $\Lambda$  with a scalar field might seem not a fresh option. Also the independence from I.C., thanks to the presence of a field attractor, is not new.

In respect to standard quintessence theories, however, SCDEW models are favoured due to a few strong points: (i) They are apparently independent from the choice of any specific potential for the  $\Phi$  field. (ii) The quintessential  $\Phi$  field is shown to have necessarily been a significant cosmic component since long time, possibly since the end of inflation. (iii) Again, the attractor solution it fulfills is not linked to any potential choice. (iv) The possibility that DE and inflaton are the same field  $\Phi$  can also be pursued: in effect, since inflation,  $\Phi$  had just a logarithmic growth matching, e.g., the logarithmic evolution of constants in a Coleman–Weinberg–like potential, so able to provide potential energy both then and now.

The idea that  $\Lambda$ CDM is such a successfull model because it *mimics* an underlying more complex cosmology is not completely new. For instance, Amendola et al. (2008) investigated cosmologies where DE arises because of the increase of neutrino mass, aiming to a scenario substantially indistinguishable from  $\Lambda$ CDM. Any such cosmology requires the introduction of extra parameters. In SCDEW models 2 extra parameters (apart of the delay  $d$ ) are needed: the mass of the WDM particle and the coupling  $\beta$ . However, if we vary these parameters within a reasonable range, we obtain cosmologies which could also be *mimiced* by sorts of  $\Lambda$ CDM with a different choice of its basic parameters. Moreover, the mass  $m_w$  could soon become a non-free parameter, if a particle candidate is found. In this case, the constraints on  $\beta$ , deriving from fluctuation amplitude, CMB spectrum, density parameter choice, etc., as illustrated in this paper, are decisive.

## ACKNOWLEDGMENTS

S.A.B. thanks C.I.F.S. (Consorzio Interuniversitario per la Fisica Spaziale) for its financial support.

## REFERENCES

- Amendola, L. 1999, Phys. Rev. D, 60, 043501  
 Amendola, L.; Baldi M., & Wetterich, C. 2008, Phys. Rev. D, 78, 023015  
 Amendola, L., & Tocchini-Valentini, D. 2002, Phys. Rev. D, 66, 043528  
 Baldi, M., Pettorino, V., Robbers, G., & Springel, V. 2010, MNRAS, 403, 1684  
 Baldi, M. 2012, MNRAS, 422, 1028  
 Bonometto S. A., Sassi G., La Vacca G., 2012, JCAP, 8, 015 (BSLV12)  
 Bonometto S. A., Mainini R., 2014, JCAP, 3, 038 (BM14)  
 Das, S., Corasaniti, P. S., & Khoury, J. 2006, Phys. Rev. D, 73, 083509  
 Ellis, J., Kalara, S., Olive, K. A., & Wetterich, C. 1989, Physics Letters B, 228, 264  
 Joachimi, B., & Bridle, S. L. 2010, A&A, 523, AA1  
 Kristiansen, J. R., La Vacca, G., Colombo, L. P. L., Mainini, R., & Bonometto, S. A. 2010, NewA, 15, 609  
 Li, B., & Barrow, J. D. 2011, Phys. Rev. D, 83, 024007  
 La Vacca, G., Kristiansen, J. R., Colombo, L. P. L., Mainini, R., & Bonometto, S. A. 2009, JCAP, 4, 007  
 Lopez Honorez, L., Reid, B. A., Mena, O., Verde, L., & Jimenez, R. 2010, JCAP, 9, 029  
 Macciò, A. V., Quercellini, C., Mainini, R., Amendola, L., & Bonometto, S. A. 2004, Phys. Rev. D, 69, 123516  
 Macciò, A. V., Paduroiu, S., Anderhalden, D., Schneider, A., & Moore, B. 2012, MNRAS, 424, 1105  
 Mainini, R., & Bonometto, S. A. 2004, Physical Review Letters, 93, 121301  
 Navarro, J. F., Frenk, C. S. & White, S. D. M., 1997, ApJ, 490, 493  
 Oh, S.-H., de Blok, W. J. G., Brinks, E., Walter, F., & Kennicutt, R. C., Jr. 2011, AJ, 141, 193  
 Parker, L. 1969, Physical Review, 183, 1057  
 Parker, L. 1971, Phys. Rev. D, 3, 346  
 Pettorino, V., Amendola, L., Baccigalupi, C., & Quercellini, C. 2012, Phys. Rev. D, 86, 103507  
 Puchwein, E., Baldi, M., & Springel, V. 2013, MNRAS, 436, 348  
 Walker, M. G., & Peñarrubia, J. 2011, ApJ, 742, 20  
 Wetterich C., 1995, A&A, 301, 321  
 Xia J.-Q., 2013, JCAP, 11, 022

APPENDIX A: PHYSICAL OPTIONS FOR  $\beta$  FADING

In the CDM– $\Phi$  coupling  $C = b/m_p$  one can straightforwardly introduce a  $\Phi$  dependence, yielding  $C(\Phi)$ ; Mainini & Bonometto (2004), e.g., took  $C = b/\Phi$ . In the context of SCDEW modes, owing to the progressive  $\Phi$  increase, instead of taking  $\beta \propto (a/a_{dg})^\alpha$ , we can assume  $\beta \propto (\Phi/\Phi_{dg})^{\tilde{\alpha}}$ ; an expression relating  $a_{dg}$  and  $\alpha$  to  $\Phi_{dg}$  and  $\tilde{\alpha}$  is then obtainable, so to achieve similar results.

A smarter option, perhaps, still keeping to field theory, amounts to replacing the lagrangian (6) in Section 2 by

$$\mathcal{L}_m = -(\mu f(\Phi/m) + \tilde{\mu})\bar{\psi}\psi. \quad (A1)$$

This modifies the field and  $\rho_c$  equations (5) into

$$\ddot{\Phi} + 2\frac{\dot{a}}{a}\Phi = -a^2V' + \rho_c\frac{Ca^2}{1 + \exp(C\Phi)\tilde{\mu}/\mu} \quad (A2)$$

$$\dot{\rho}_c + 3\frac{\dot{a}}{a}\rho_c = -\rho_c\frac{C\dot{\Phi}}{1 + \exp(C\Phi)\tilde{\mu}/\mu}$$

where we keep to the expression of  $C = b/m_p$  with constant  $b$ . Until  $b\Phi \ll m_p$  coupling is as usual. But, when  $\Phi$  approaches the Planck scale, the effective coupling is cut off.

A further option, not affecting field theory, can also be envisaged, although its effectiveness depends on a number of still open problems. As a matter of fact one can wonder what happens to CDM nonlinearities after they entry in the horizon. The question is whether the nonlinear gravitational growth of CDM fluctuations ends up into a relativistic collapse or into virialized structures. In the former case, one should not forget that the action of  $\Phi$  on CDM can be described by replacing  $G$  with  $G^*$ , adding the antiderivative term, etc., only until a non relativistic approximation holds. On the contrary, the dynamics of a relativistic collapse, in the presence of  $\Phi$  interactions, is still an open problem.

We can however formulate two conjectures: (i) The nonlinear gravitational growth ends up into virialization if the CDM matter density overcomes a suitable threshold  $\bar{\rho}_c$ . (ii) If relativistically collapsed CDM structures form, the  $\Phi$  field is unable to interact with CDM there inside.

Within this context, let us imagine to treat the growth of a spherical density fluctuation, starting from its entry in the horizon, when it involves fluctuations of amplitude  $\Delta_c \sim \bar{\Delta} \sim 10^{-5}$  (for CDM and the other cosmic components, respectively). Let  $R = ac$  be its radius,  $a$  being the scale factor. At the conformal initial time  $\tau_i$ , the radius is  $R_i = a_i c_i \sim \tau_i$  with  $\dot{c}_i = 0$ . Let then  $x = c/c_i$ ; at any  $\tau > \tau_i$ ,  $x$  shall fulfill the equation

$$\ddot{x} = -\frac{1}{2}\left[\frac{\Delta_c \gamma \Omega_c}{x^2} + 2(1 - \Omega_c)\bar{\Delta}x\right]\frac{1}{\tau^2}, \quad (A3)$$

almost exact in the period characterized by self-similar expansion. The equation is more complex later on, but the key features are the same. In particular, let us remind that  $\Omega_c = 1/2\beta^2$ ,  $\gamma = 1+4\beta^2/3$ .

According to this equation, as expected, the CDM enhancement expands, enters a non-linear regime, and then starts to recontract, while the other components still undergo a linear evolution. Recontraction ends up into a complete gravitational collapse only if the Schwarzschild density

$$\rho_s \simeq 3 \times 10^{16}(\text{g/cm}^3)(M/M_\odot)^{-2}$$

is attained before arriving to the critical density  $\bar{\rho}_c$ . Here  $M$  is the mass involved in the collapse. Accordingly, at early times, CDM non linearities can be described as a rippled CDM distribution. There will however be a time after which CDM starts to form sorts of Black Holes (BH).

In the late epochs, therefore, CDM has mostly turned into a suitable BH distribution, ceasing its interaction with  $\Phi$ .

This paper has been typeset from a  $\text{T}_{\text{E}}\text{X}$ /  $\text{L}_{\text{A}}\text{T}_{\text{E}}\text{X}$  file prepared by the author.